AQA Computer Science A-Level

4.3.5 Sorting algorithms

Intermediate Notes

Specification:

4.3.5.1 Bubble sort

Know and be able to trace and analyse the time complexity of the bubble sort algorithm.

4.3.5.2 Merge sort

Be able to trace and analyse the time complexity of the merge sort algorithm.

Sorting Algorithms

In the case of a sorting algorithm, the task is to put the elements of an array into a specific order. A sorted list can often be more useful than an unsorted one, for example: the binary search algorithm can only be used on a sorted list.

There are multiple different sorting algorithms of varying complexity. The two investigated below are the bubble sort and the merge sort.

Synoptic Link

O(logn) and **O(n)** are examples of **Big O notation**. Big O is a way of **classifying algorithms** based on **time and space complexity**. In this case, O(n) is more complex than O(logn).

Big O is covered in **Classification of Algorithms** under **Fundamentals of Algorithms**.

Sorting Algorithms Overview:

To be sorted into ascending order: 12 3 8

Bubble Sort

Make passes through the data and swap adjacent items. Stops passing through data when no swaps are performed. BigO is $O(n^2)$

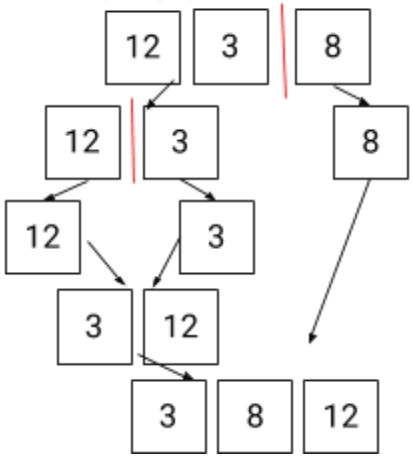
Pass 1: 12 3 8 → 3 12 8 → 3 8 12

Pass 2: 3 8 12 → 3 8 12 → 3 8 12

Sorted list: 3 8 12

Merge Sort

'Divide and Conquer' Method - split array up into individual sorted lists and then merge them together. BigO is O(nlogn)



Bubble Sort

The bubble sort algorithm swaps the position of adjacent items to order them. It has a time complexity of $O(n^2)$ which makes it relatively inefficient.

Bubble Sort Example 1:

The following array needs to be sorted into ascending alphabetical order.

Position	0	1	2	3	4
Data	Freddie	Brian	Roger	Adam	John

The first step is to compare the first two pieces of data.

Position	0	1	2	3	4
Data	Freddie	Brian	Roger	Adam	John

Freddie > Brian. Therefore Freddie and Brian should swap places.

Position	0	1	2	3	4
Data	Brian	Freddie	Roger	Adam	John

Position 1 is now checked against position 2.

Position	0	1	2	3	4
Data	Brian	Freddie	Roger	Adam	John

Freddie < Roger. Hence they are in the correct order, and do not need to be swapped.

Position	0	1	2	3	4
Data	Brian	Freddie	Roger	Adam	John

The data in position 2 of the array is checked against the data in position 3.

Position	0	1	2	3	4
Data	Brian	Freddie	Roger	Adam	John

Roger > Adam. They should swap positions.

Position	0	1	2	3	4
Data	Brian	Freddie	Adam	Roger	John

The third and fourth positions are checked.

Position	0	1	2	3	4
Data	Brian	Freddie	Adam	Roger	John

Roger > John, so they should swap positions in the array.

Position	0	1	2	3	4
Data	Brian	Freddie	Adam	John	Roger

There are no more positions to check. We can now say that we have made one pass through the data. We also know that the data in the last position, “Roger”, is in the correct position - we will highlight this in green to show that it does not need to be checked again; a good bubble sort algorithm will not have to check the last position. From looking, we can also see that “John” is in the correct position, however a computer will not know this until the second pass has been made.

The first two positions are checked again.

Position	0	1	2	3	4
Data	Brian	Freddie	Adam	John	Roger

Brian < Freddie. They are ordered, so should not be swapped.

Position	0	1	2	3	4
Data	Brian	Freddie	Adam	John	Roger

Position 1 is checked against position 2.

Position	0	1	2	3	4
Data	Brian	Freddie	Adam	John	Roger

Freddie > Adam. These pieces of data should swap positions.

Position	0	1	2	3	4
Data	Brian	Adam	Freddie	John	Roger

The second and third positions are checked.

Position	0	1	2	3	4
Data	Brian	Adam	Freddie	John	Roger

Freddie < John, so they do not have to be swapped.

Position	0	1	2	3	4
Data	Brian	Adam	Freddie	John	Roger

We can now say that we have made two passes through the data. Now John is definitely in the correct position, so it will be locked down, and there is no need to check it on the next pass.

The first two positions are checked again.

Position	0	1	2	3	4
Data	Brian	Adam	Freddie	John	Roger

Brian > Adam. They should swap positions.

Position	0	1	2	3	4
Data	Adam	Brian	Freddie	John	Roger

We can see that the data is correctly ordered, but a computer has no way of telling this. It can only determine that the list is in the correct order if it makes a pass through the data with no swaps.

The data in position 1 and 2 are checked.

Position	0	1	2	3	4
Data	Adam	Brian	Freddie	John	Roger

Brian < Freddie. They do not need to be swapped.

Position	0	1	2	3	4
Data	Adam	Brian	Freddie	John	Roger

We have now made a third pass through the data. The algorithm knows that the item in position 2, “Freddie” is in the correct place.

The data in position 0 and 1 are checked against each other.

Position	0	1	2	3	4
Data	Adam	Brian	Freddie	John	Roger

Adam < Brian. They do not need to be swapped. The data is now in the correct order.

Bubble Sort Example 2

A question may ask you how the data looks after a stated number of passes, or how many passes are required to sort the array. The below example only shows how the list would look after each pass.

Here is our unsorted list:

Position	0	1	2	3	4	5	6
Data	Mon	Tues	Weds	Thurs	Fri	Sat	Sun

First Pass:

Position	0	1	2	3	4	5	6
Data	Mon	Tues	Thurs	Fri	Sat	Sun	Weds

Second Pass:

Position	0	1	2	3	4	5	6
Data	Mon	Thurs	Fri	Sat	Sun	Tues	Weds

Third Pass:

Position	0	1	2	3	4	5	6
Data	Mon	Fri	Sat	Sun	Thurs	Tues	Weds

Fourth Pass:

Position	0	1	2	3	4	5	6
Data	Fri	Mon	Sat	Sun	Thurs	Tues	Weds

Although the data is sorted, the computer needs to make a pass through the data where there are no swaps.

Fifth Pass:

Position	0	1	2	3	4	5	6
Data	Fri	Mon	Sat	Sun	Thurs	Tues	Weds

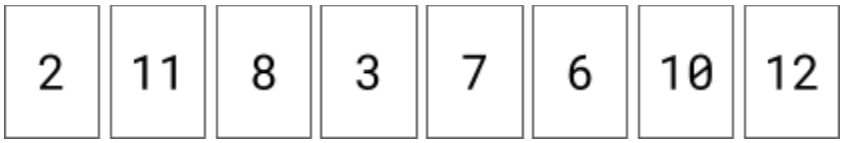
This list took 5 passes through the data to be sorted.

Merge Sort

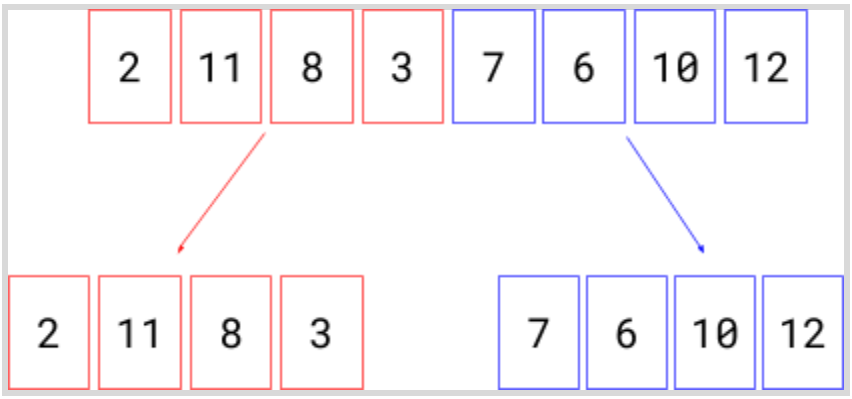
A merge sort orders arrays by splitting them into smaller lists, and then reforming them - the 'divide and conquer' method. It is quicker than a bubble sort; it has a time complexity of O(n log n).

Merge Sort Example 1:

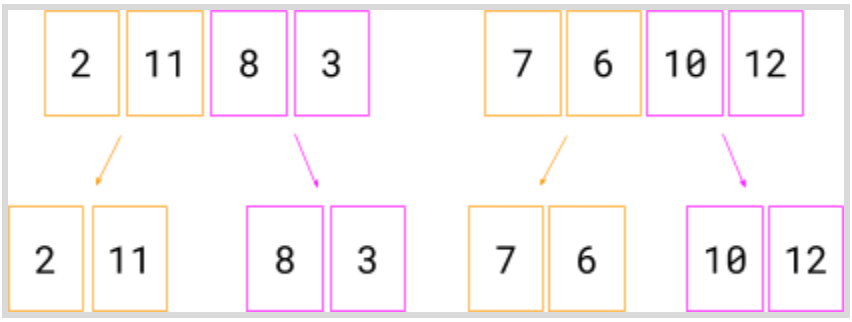
Here is an unsorted list.



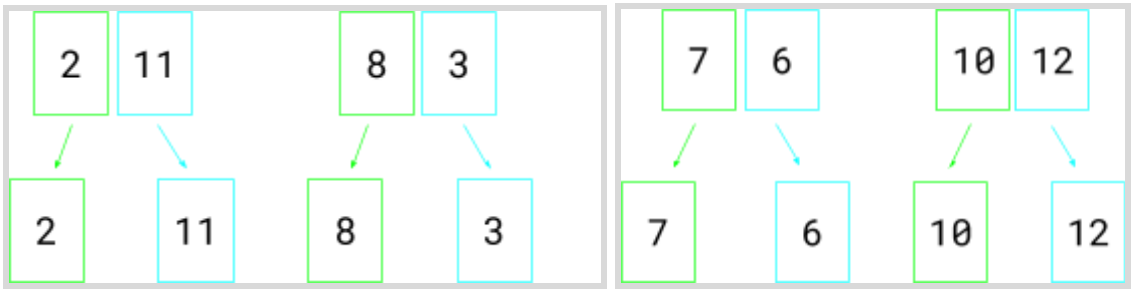
The first stage in a merge sort is to split the list into two smaller lists.



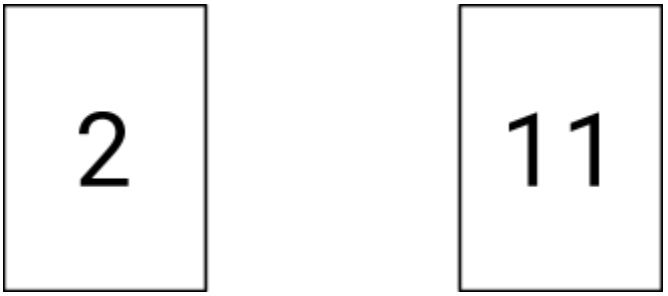
These lists are still unsorted, so they need to be split further.



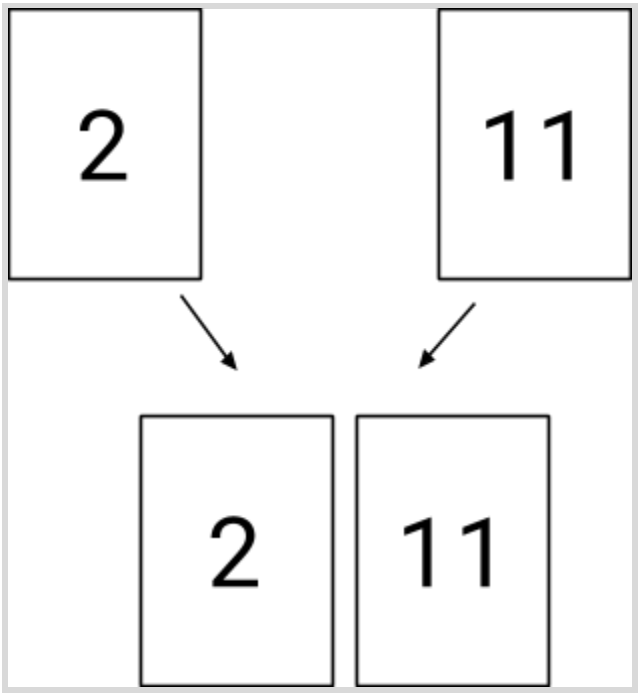
These lists are unsorted, with two elements; they need to be split further.



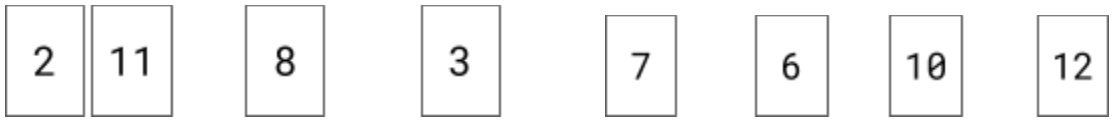
Now there are eight lists each with one element. Since there is only one element in each, they are all ordered lists. Now they can be put back together by comparison. We start with the first two lists.



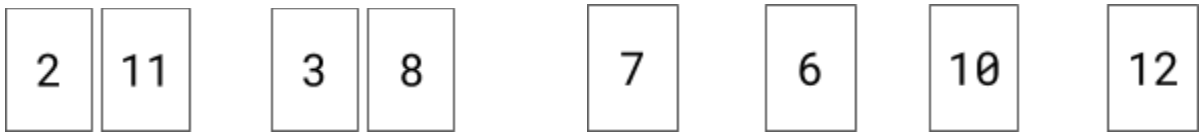
2 < 11, so the ordered list is 2, 11.



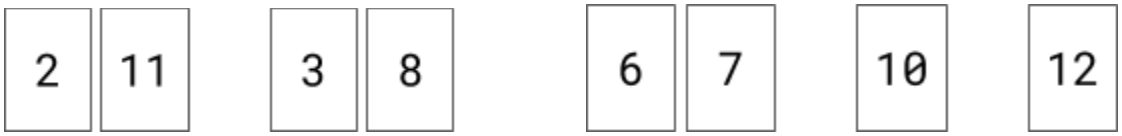
Our collection of lists looks like this:



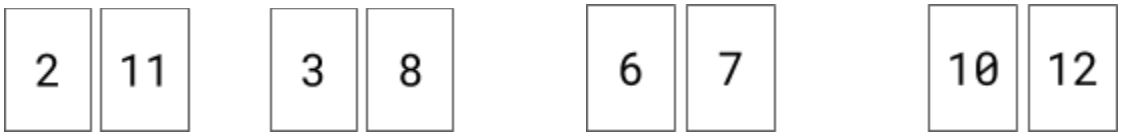
The next pair is 8 and 3. 8 > 3 so they are paired up like this.



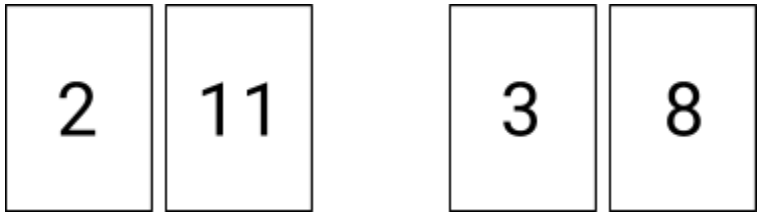
The next pair is 7 and 6. 7 > 6, so they are combined with 6 as the first element in the list.



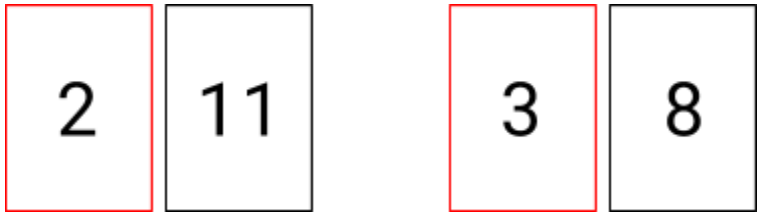
The last pair is 10 and 12. 10 < 12, so they are paired up as follows.



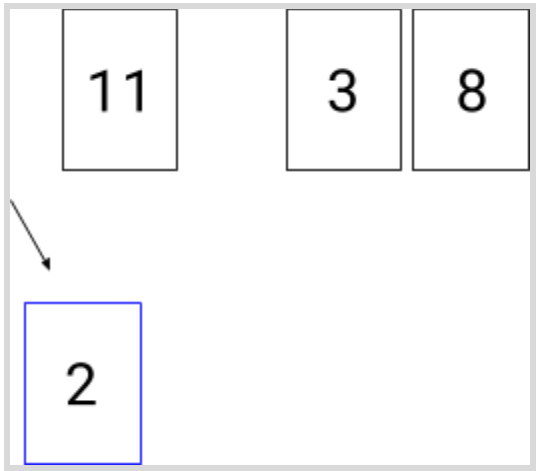
We now have four sorted lists, each containing two elements. The next stage is to once again consider adjacent lists. We start with the first two lists, (2,11) and (3,8).



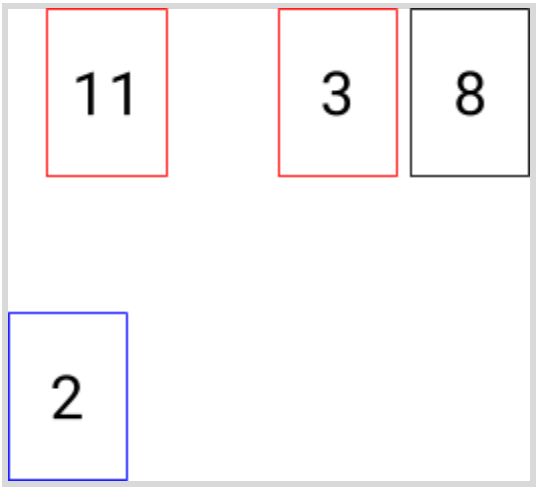
The smallest element in the first list is 2, and the smallest element in the second list is 3.



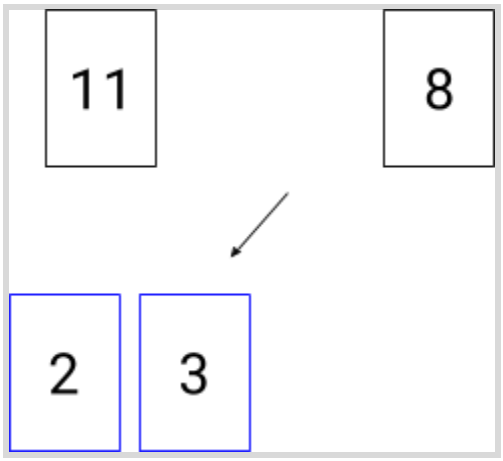
2 < 3, so it is added to the new list first.



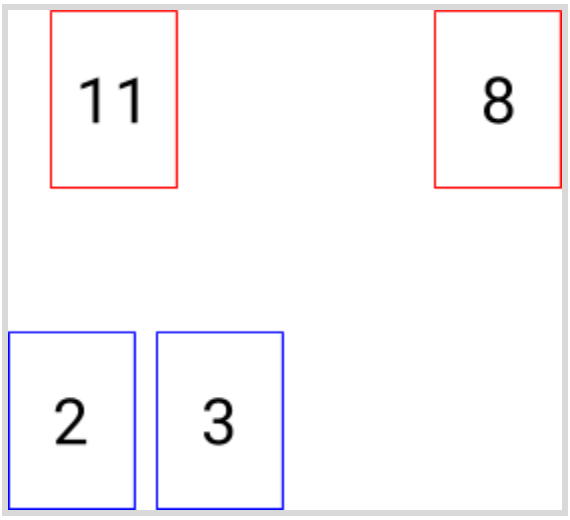
Now the smallest element in the first list is 11.



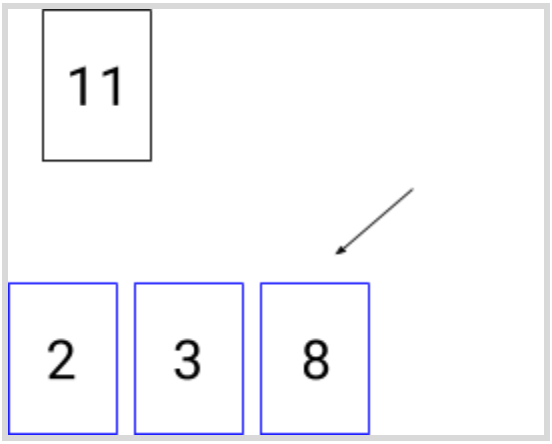
3 < 11, so it is added to the list next.



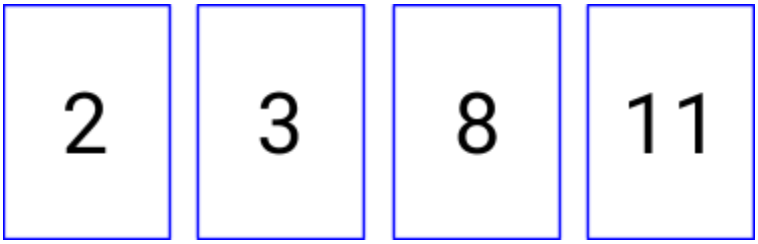
Now the smallest element in the second list is 8.



8 < 11, so it is added to the list first.



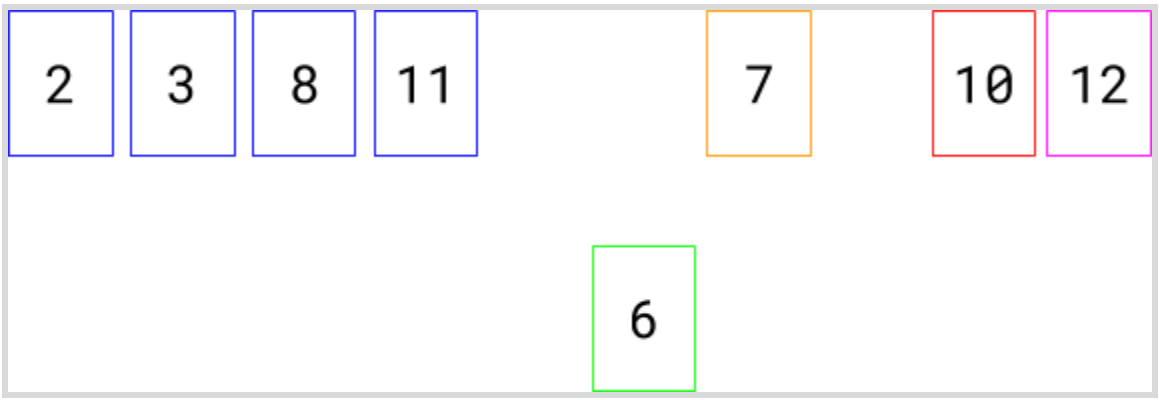
The final element is 11, so it goes on the end of the sorted list.



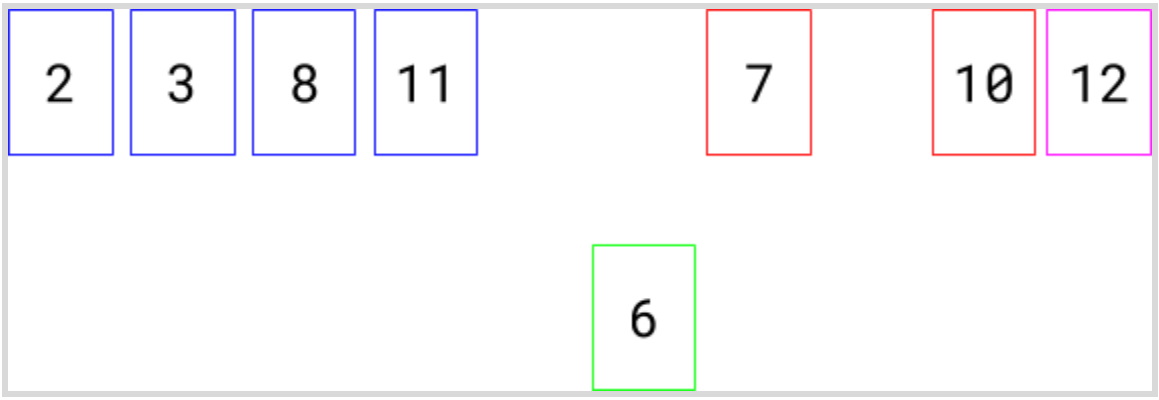
All of our lists together look like this.

We now need to put together the other two lists. The smallest element in the orange list is 6, and the smallest element in the pink list is 10.

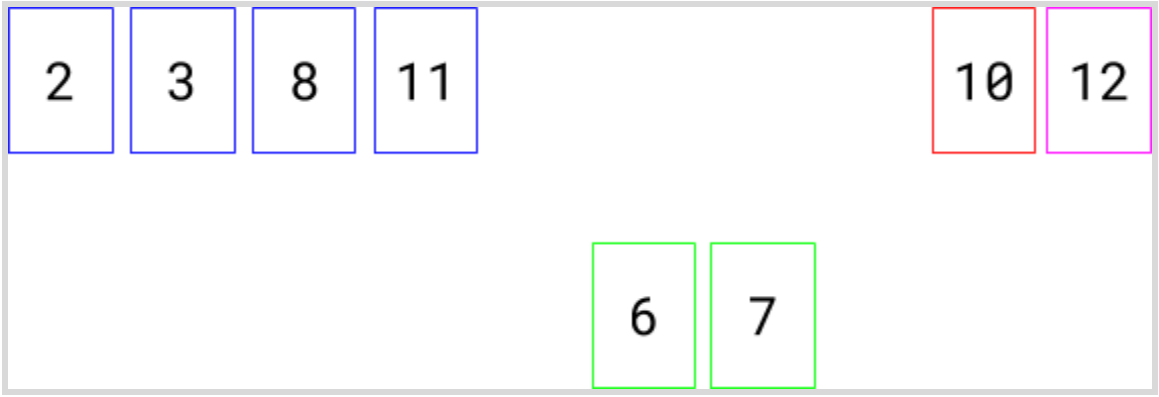
6 < 10 so it is added to the new sorted list first.



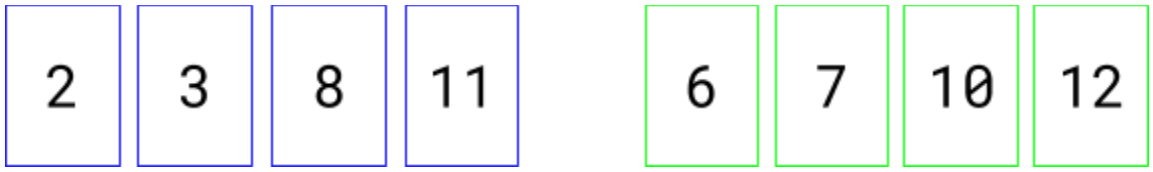
7 is now the smallest element in the orange list.



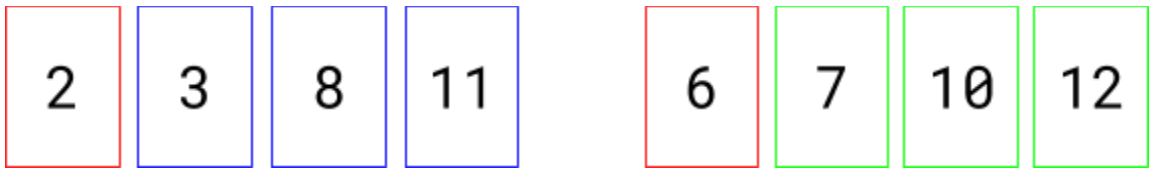
7 < 10 so 7 is added first.



The pink list is already sorted, so it can be added onto the end of the 6 and 7.

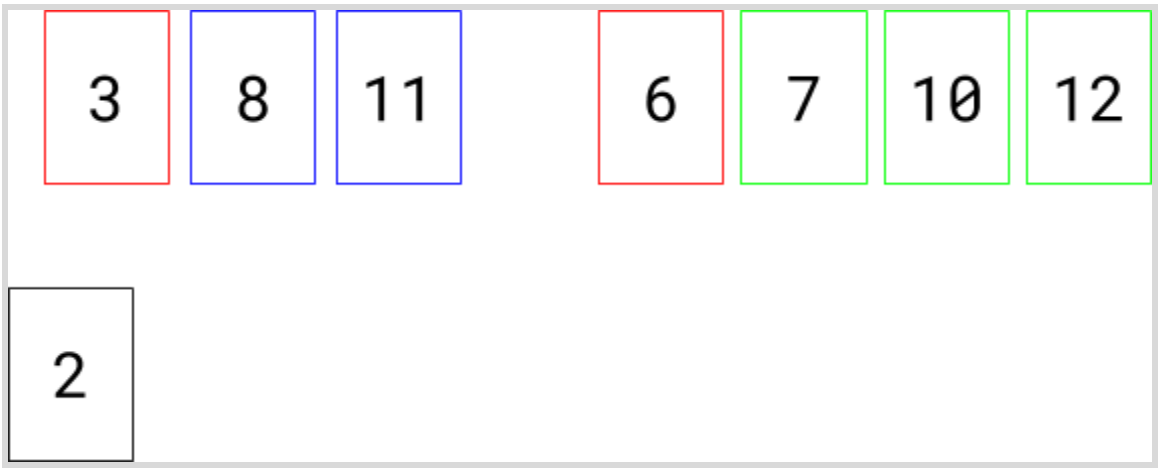


We have two sorted lists each of four elements. The process of combining lists is repeated. The smallest element in the blue list is 2, and the smallest element in the green list is 6.

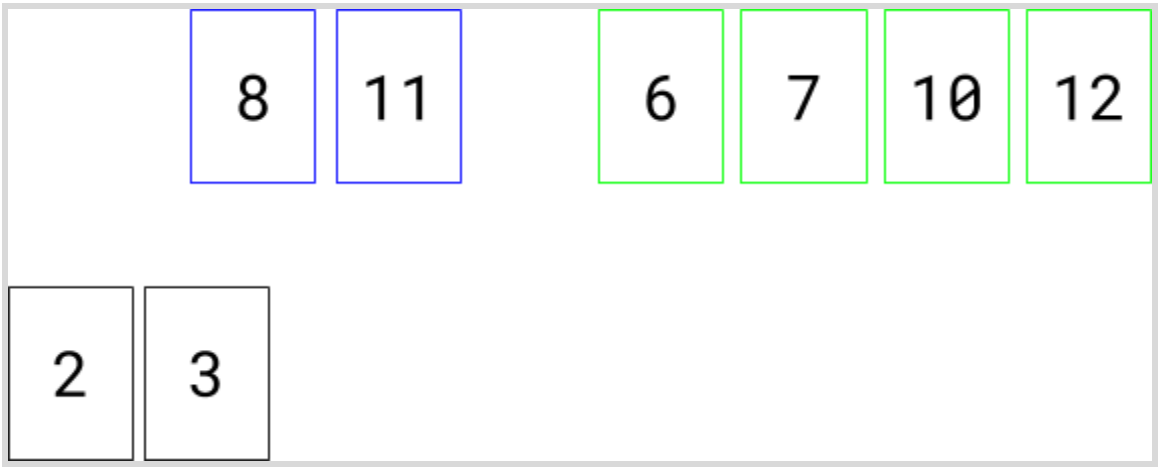


2 < 6. 2 is first on the sorted list.

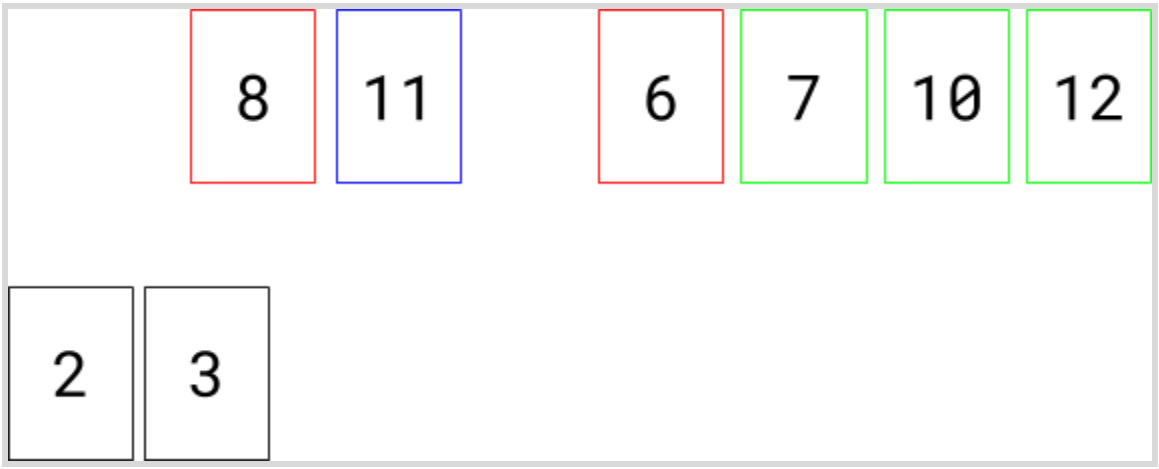
The smallest element in the blue list is 3.



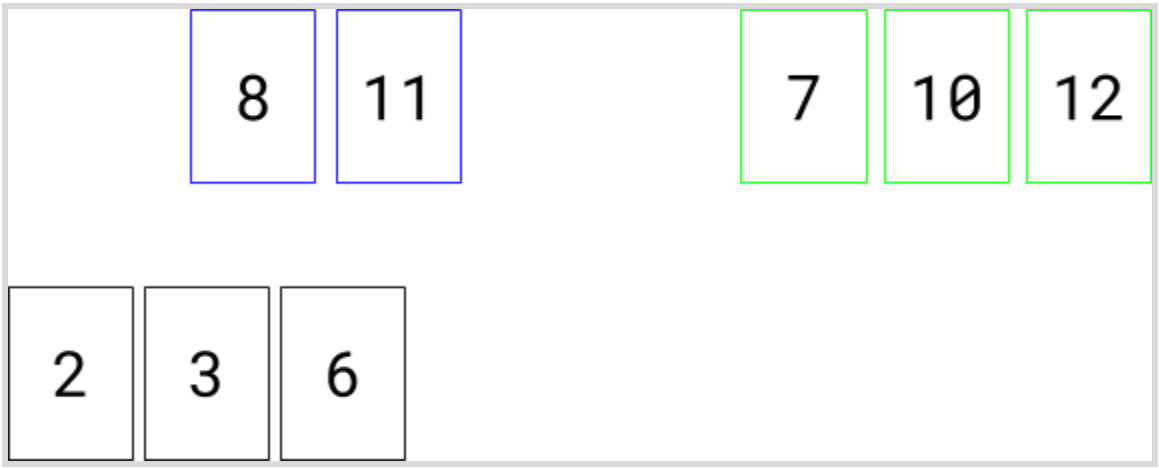
3 < 6, so 3 the next element on the sorted list.



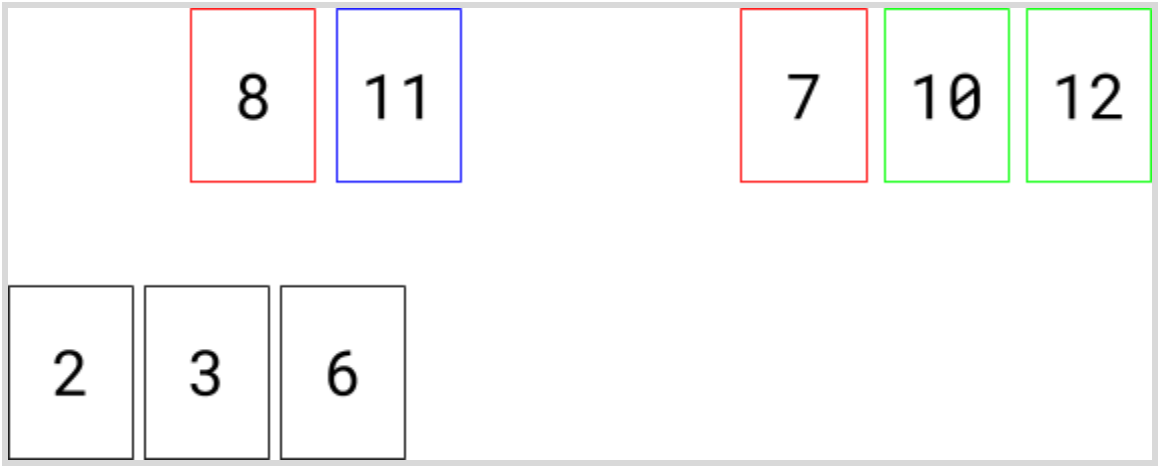
The smallest element in the blue list is 8.



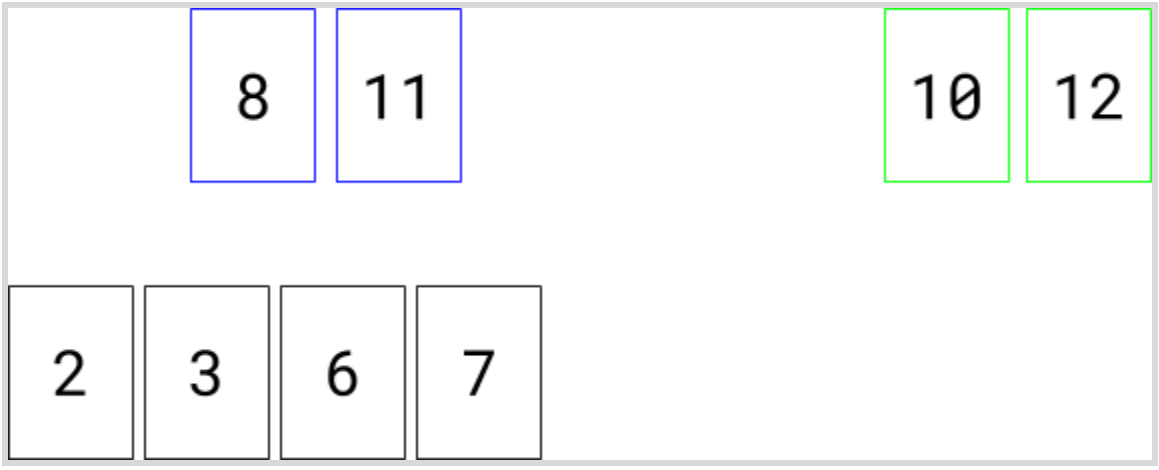
6 < 8, so 6 is added next.



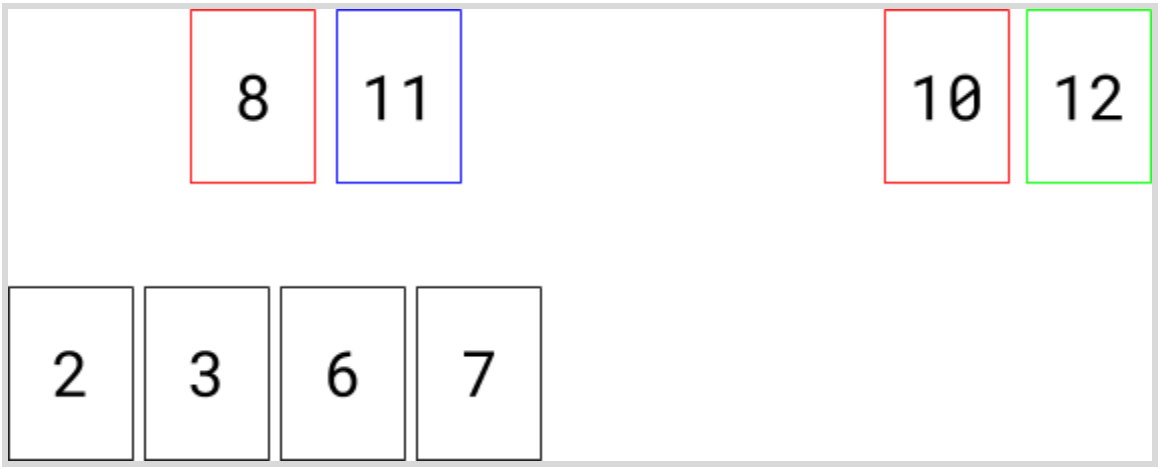
The smallest item in the green list is 7.



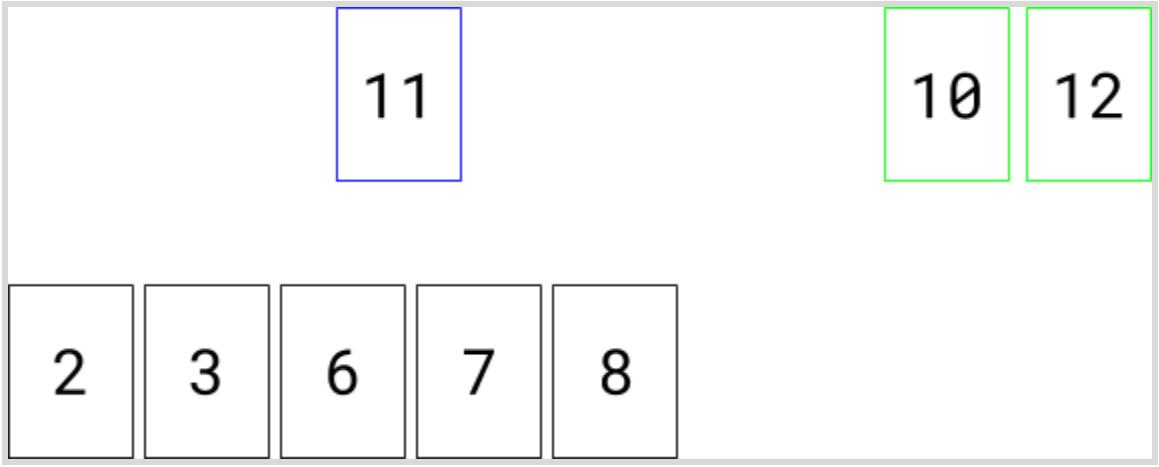
8 > 7. Hence, 7 is next on the list.



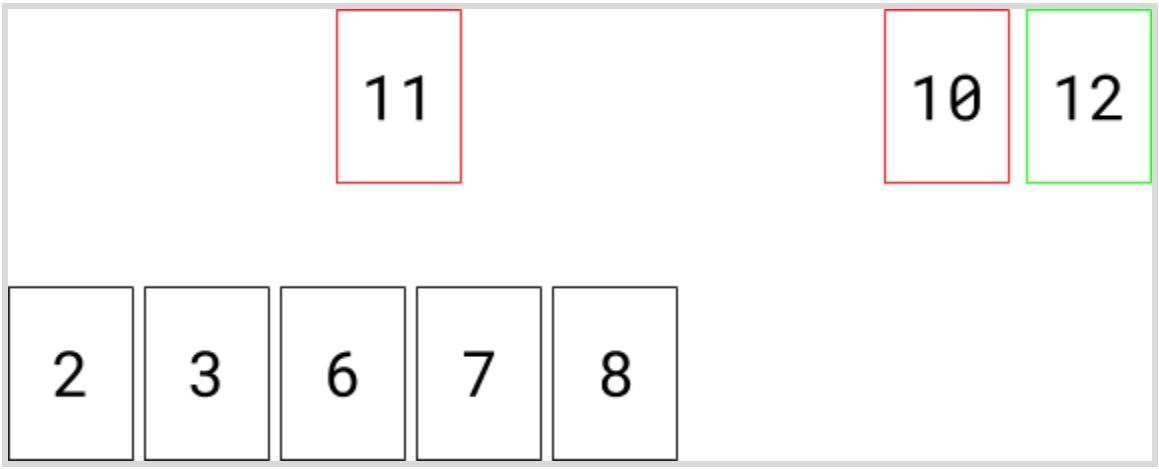
The smallest element in the green list is 10.



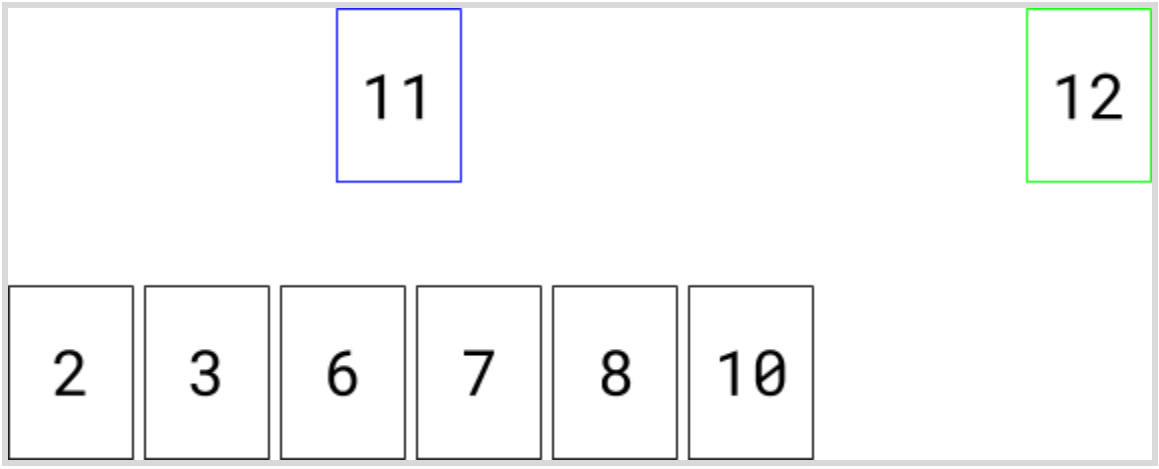
8 < 10. 8 is added next.



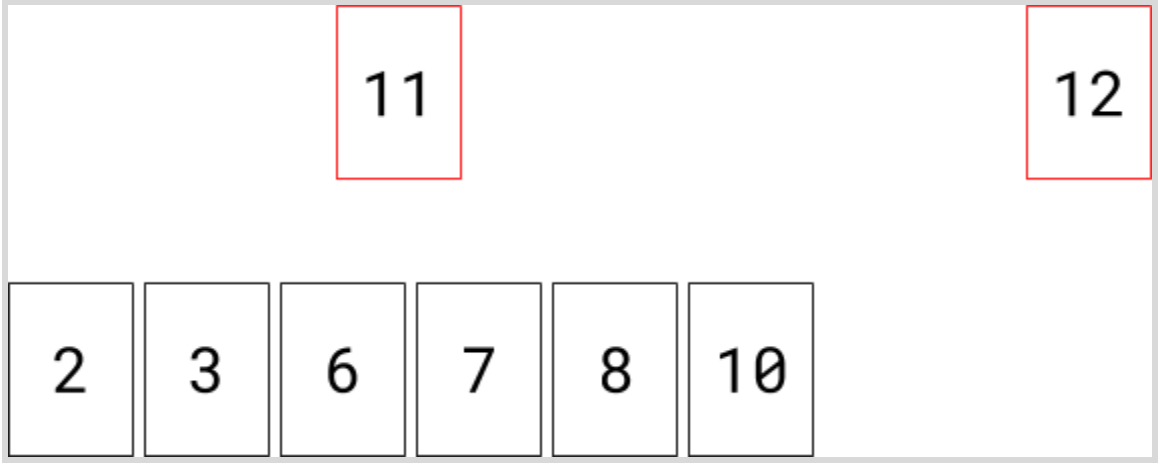
The smallest item in the blue list is 11.



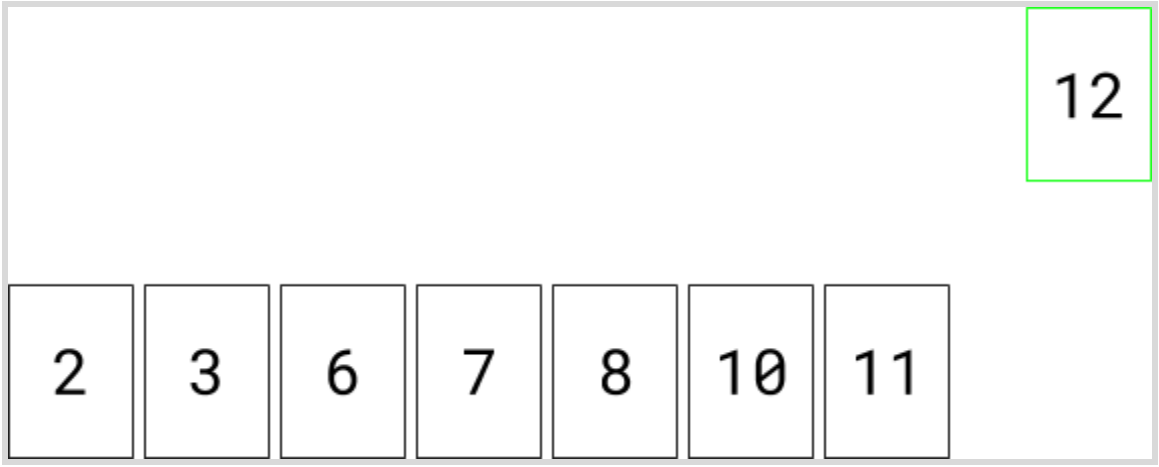
11 > 10. Thus, 10 is added next.



12 is the smallest element in the green list.



11 < 12. 11 is added to the list.



The blue list is empty, so the contents of the ordered green list can be added to the end of the black list. This is our final ordered list.

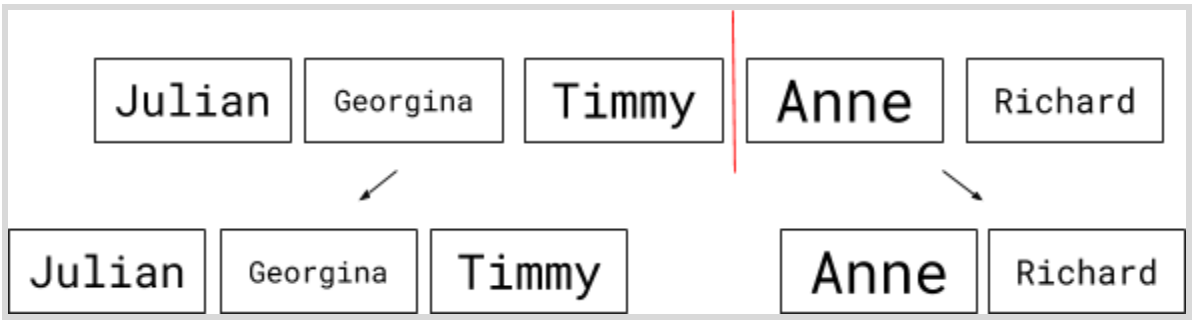
2	3	6	7	8	10	11	12

Merge Sort Example 2:

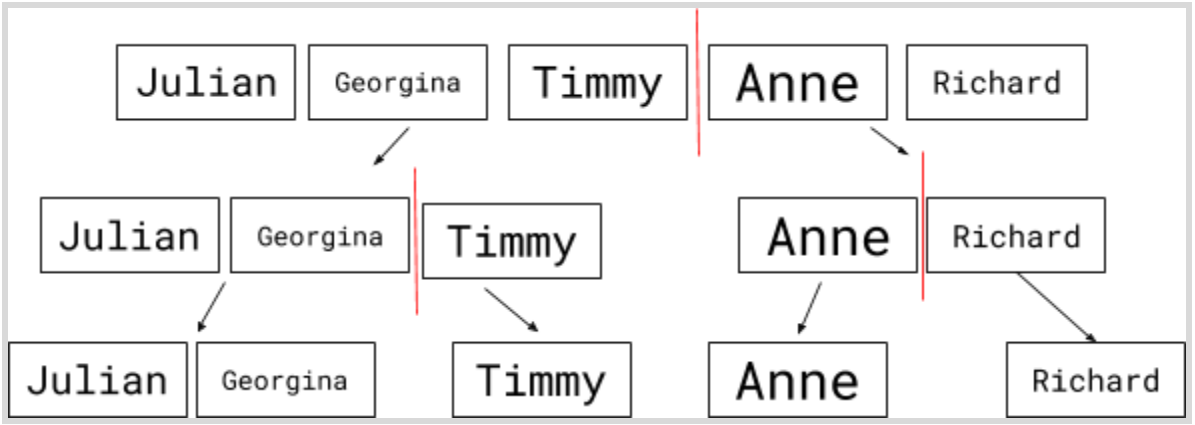
Here is an unsorted array:

Julian	Georgina	Timmy	Anne	Richard

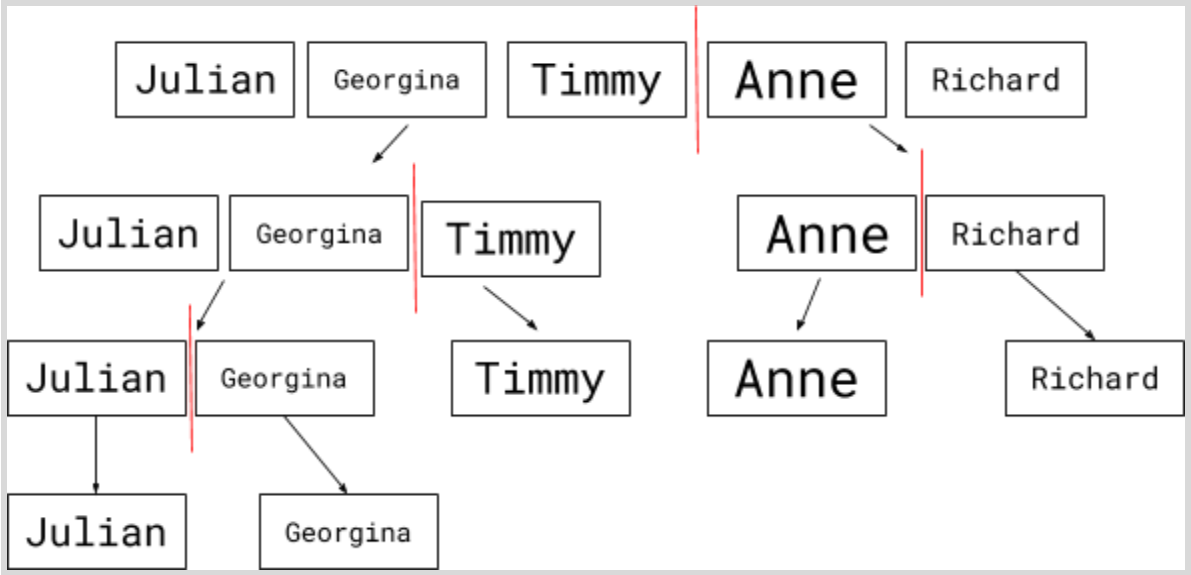
The first step is to split the array into two.



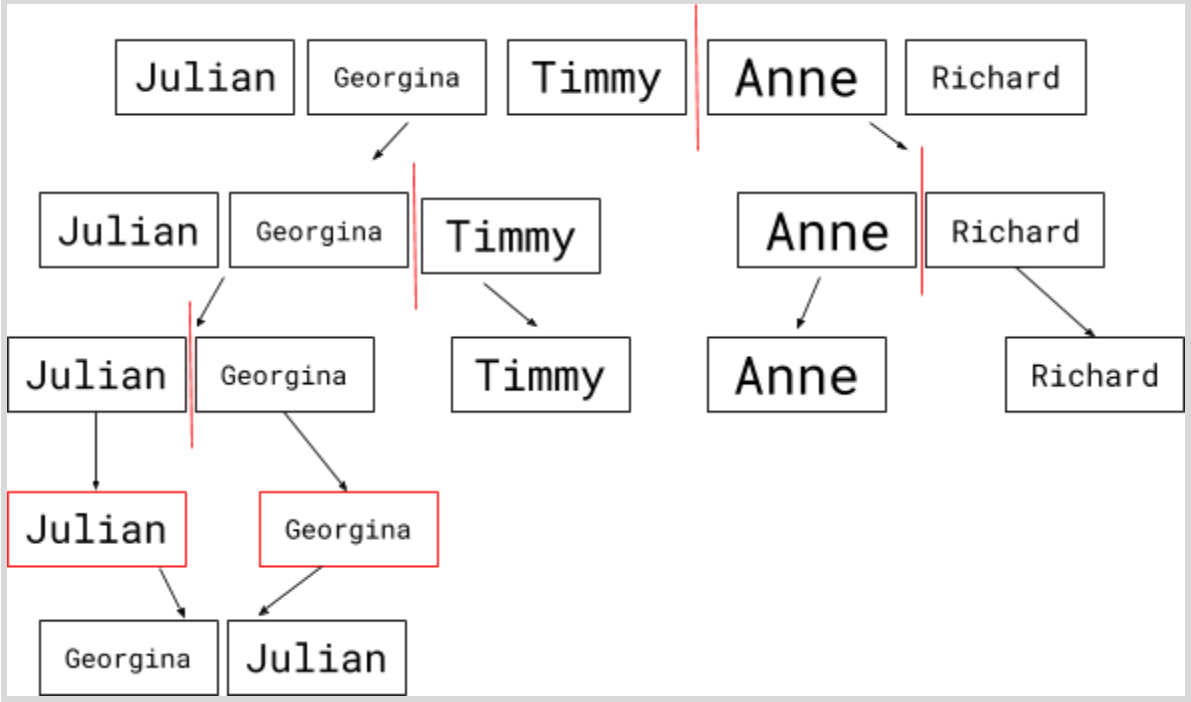
Split them again.



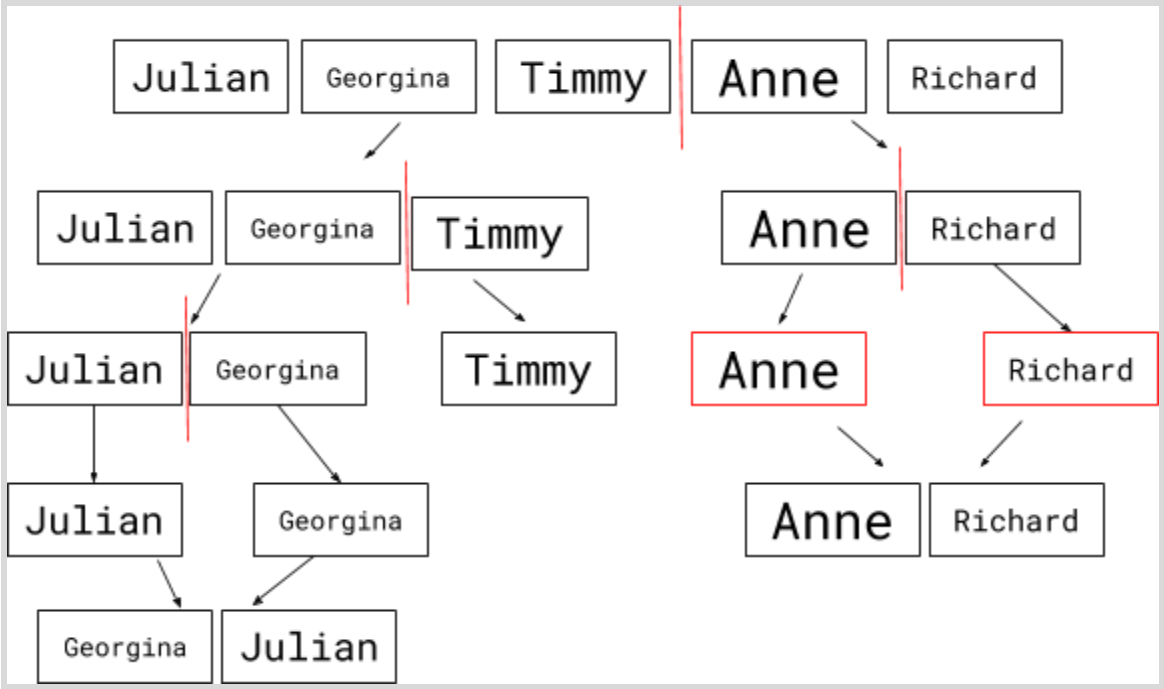
Julian and Georgina are still a pair, so they need to be split again.



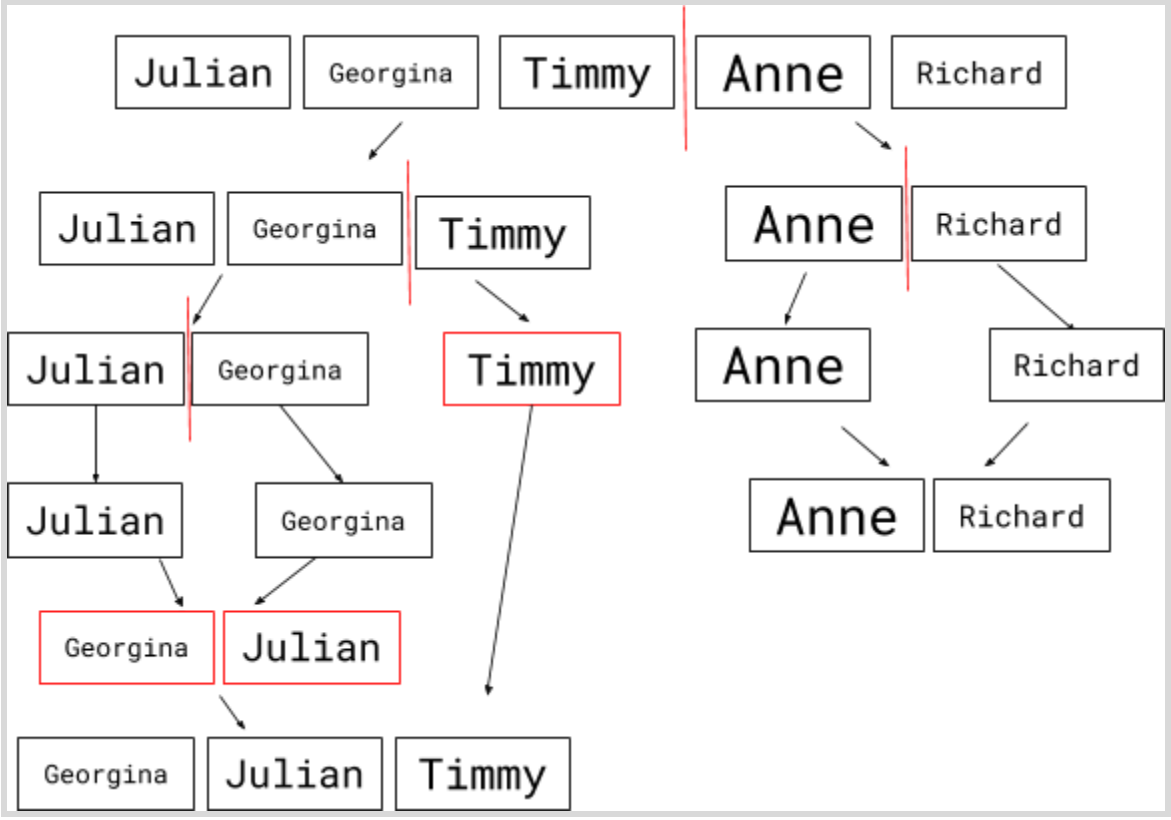
Reform the lists by merging ordered single items. Julian & Georgina:



Reform the lists by creating ordered pairs. Anne & Richard:



Reform lists with further comparisons.



Finally, merge both sorted lists together.